Degeneration on Controllability and Activity

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Outline

Degeneration on Controllability

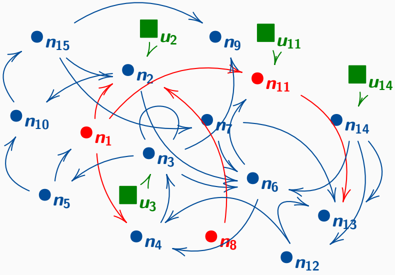
Degeneration on Spiking Neural Networks

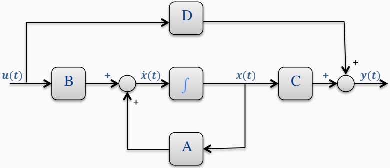
Structure-Dynamics Correlation

Degeneration on Controllability

Controllability of brain networks

Neuronal networks

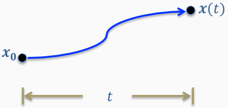




State-space model

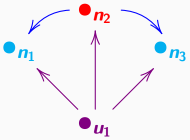
$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

- A system is **controllable** if it can be stirred to a desired target state in a finite time.

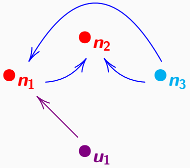


Test for Controllability

- **Kalman's controllability test** (Kalman 1960)
  A linear system, $\dot{x}(t) = Ax(t) + Bu(t)$, is controllable if its controllability matrix $\mathcal{C} = [B, AB, A^2B, \ldots, A^{n-1}B]$ has full rank.
- **Structural controllability**
  A LS is **structurally controllable** if there exists a controllable system whose binary versions of the connectivity and input projection matrices are correspondingly the same.
- **Lin's structural controllability test** (Lin 1974)
  A LS is str. controllable if it has neither **dilation** nor **inaccessibility**.



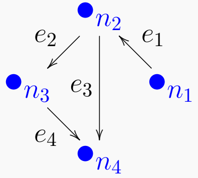
dilation



inaccessibility

Optimal Configuration of Controllable LS

- **Controllability of Complex Networks** (Liu et al. 2011, *Nature*) finding the minimum number of 'driver nodes' for a controllable LS.
  - **Matching Set**:
    $(a, b) \in M$ and $(c, d) \in M \Rightarrow a \neq c$ and $b \neq d$.
  - **Maximal Matching Set**
    $(a, b) \notin M \Rightarrow \{(a, b)\} \cup M$ is not a matching set.
  - **Maximum Matching Set**
    maximal and with maximum cardinality.
  - **Driver Nodes**
    unmatched nodes



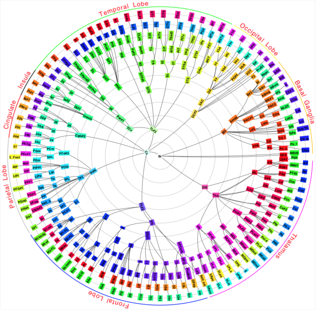
matching set: $M = \{e_1\} = \{(n_1, n_2)\}$

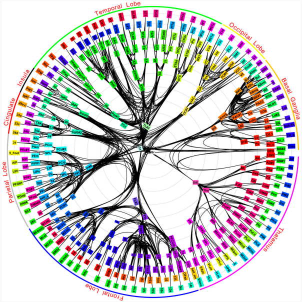
maximal mset: $M = \{e_1, e_3\}$

maximum mset: $M = \{e_1, e_2, e_4\}$

fewest drivers: $D = \{n_1\}$

Macaque brain



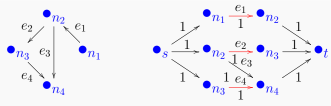


Modha et al. 2014 (adopted)

Connectivity

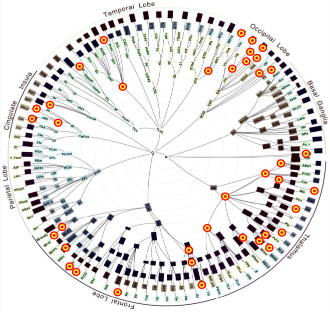
- Network size: $\boldsymbol{N = 360}$
- Network density: $\boldsymbol{\rho = 0.05}$
- num of edges: $\boldsymbol{nE = 6602}$

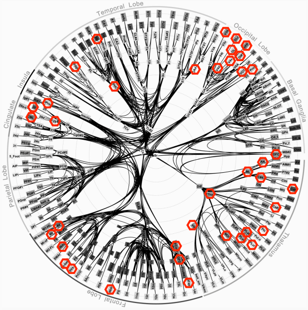
Maxflow



Of the first steps: 1.1. Identifying CNs...

A set of control nodes





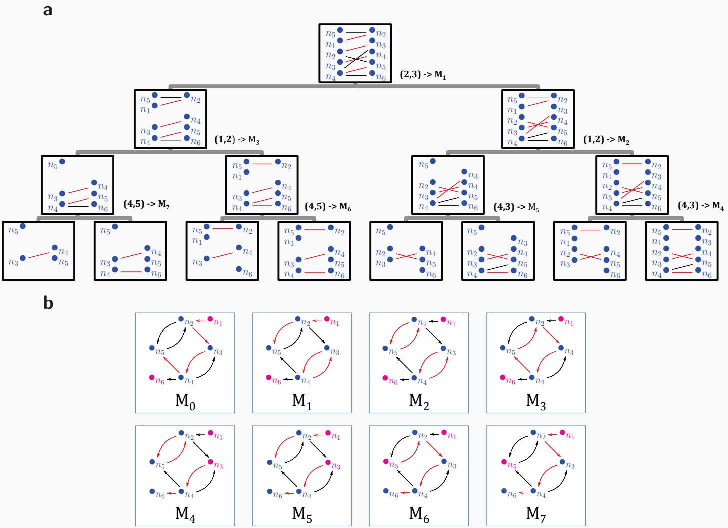
Modha et al. 2014 (adapted)

In total: 39 driver nodes (10.8% of the nodes)

Region	DiE	OL	FL	TL	Cg	Ins	MB
nCN	12	9	9	4	3	1	1

Of the first steps: 1.2. Identifying alternative MMs

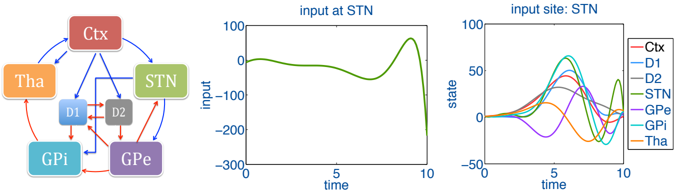
Enumerating MMs



Of the first steps: 2. Input design on simple models

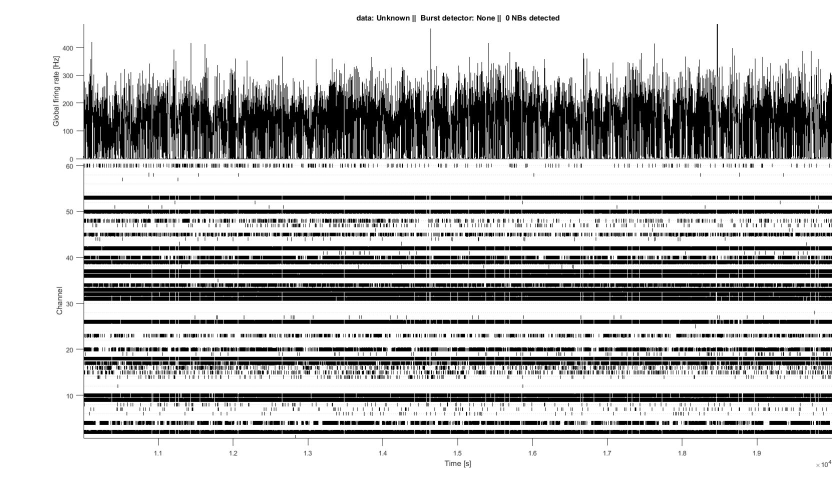
Control input design

- Equivalent Statements:
   i. The system $\dot{x} = Ax + Bu$ is controllable.
   ii. $W_{\mathcal{C}}(t) = \int_0^t e^{A\tau} BB' e^{A'\tau} d\tau$ is non-singular.
   iii. $\mathcal{C} = [B, AB, A^2B, \ldots, A^{n-1}B]$ has full rank.
   iv. $\forall \lambda_i$ of A, $\Lambda_i = [(A - \lambda_i I), B]$ has full rank.
- $u(A, B, t, x_0, x_{t_f}) = -B^T e^{A^T(t_f - t_0)} W_c^{-1}(t_f)[e^{At_f} x_0 - x_f]$
- demo. on simple BG model

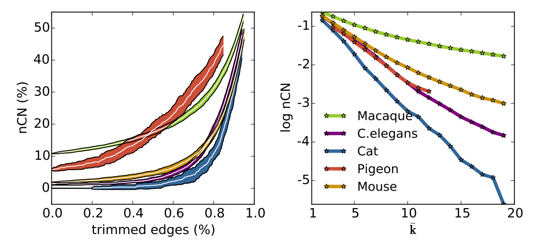


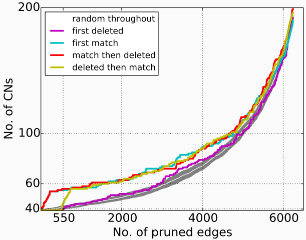
Our first steps: 3. on culture data

Spontaneous activity



Pruning on control profile





Primary targets of the pruning strategies:

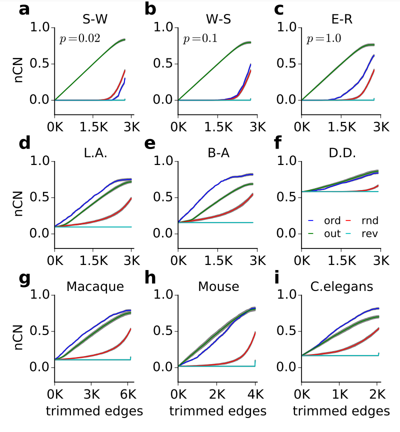
- **random pruning**: random edges
- **out/in - pruning**: outgoing/ incoming edges of a random node
- **ordered-MM pruning**: edges from MM set of earliest generation
- **resilient pruning**: edges from MM set of latest generation

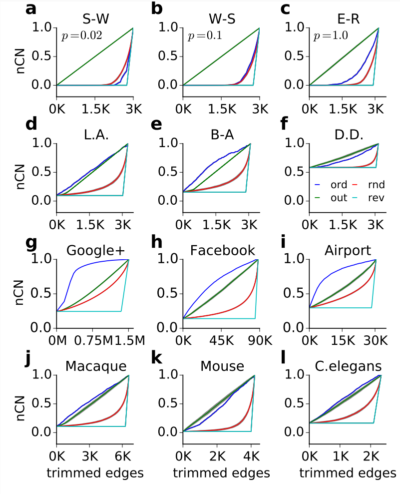
Two scenarios:

- **conditioned**: while keeping the network intact
- **unconditioned**: with no concern to fragmentation

Effect of pruning on *nCN* of networks

Conditioned vs unconditioned edge pruning

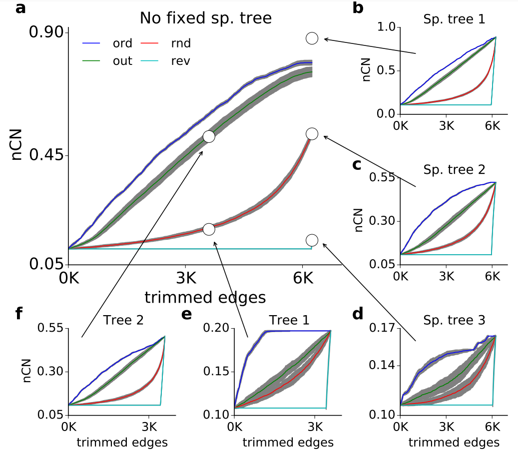




- The three biological networks resemble the scale-free networks.

Protecting spanning digraphs

Towards a common structure



- protecting a given set of edges affects the rate of change in nCN.

Analyzing different networks

a. Conditioned pruning

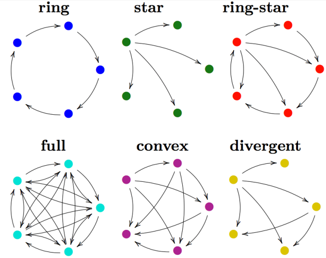
	Fig.	Network	nE	nV	$n(d<3)$	nCr	nCN_0	[$Mean$ (nCN_f)] $\pm \sigma$ ord.	out.	rnd.	rev.
W-S	2.a.	**Small-world, $p = 2\%$**	3,000	250	0	0	1	78 ± 3.5	207 ± 3.1	100 ± 3.6	22 ± 3.1
W-S	2.b.	**Watts-Strogatz, $p = 10\%$**	3,000	250	0	0	1	123 ± 3.1	198 ± 2.6	102 ± 4.3	21 ± 3.5
W-S	2.c.	**Erdös-Rényi, $p = 100\%$**	3,000	250	0	0	1	155 ± 4.4	192 ± 3.3	105 ± 4.8	5 ± 1.4
SF	2.d.	**Local attachement**	3,250	250	0	35	24	188 ± 3.0	180 ± 3.5	122 ± 3.9	24 ± 0.3
SF	2.e.	**Barabási-Albert**	3,081	250	0	28	39	205 ± 3.2	172 ± 2.9	136 ± 4.2	39 ± 0.0
SF	2.f.	**Duplication divergence**	3,151	250	1	0	146	210 ± 2.6	216 ± 4.1	166 ± 2.9	146 ± 0.0
BNN	2.j.	**Macaque**[74]	6,602	360	26	25	39	285 ± 3.3	273 ± 6.5	191 ± 4.5	51 ± 2.9
BNN	2.k.	**Mouse**[75]	4,208	213	0	6	4	175 ± 3.3	172 ± 4.6	102 ± 3.5	20 ± 2.7
BNN	2.l.	**C elegans**[12]	2,345	297	25	24	49	243 ± 3.2	208 ± 5.0	160 ± 4.0	58 ± 2.2
WS and SF (N,ρ)	S4.a.	**Random, $n = 250$ (2.c)**									
WS and SF (N,ρ)	S4.b.	**Random, $n = 500$**	12,584	500	0	0	1	335 ± 5.2	429 ± 3.6	208 ± 4.9	6 ± 2.1
WS and SF (N,ρ)	S4.c.	**Random, $n = 1000$**	50,082	1,000	0	0	1	699 ± 5.6	913 ± 4.1	416 ± 8.5	7 ± 2.3
WS and SF (N,ρ)	S4.d.	**Random, $\rho = 2\%$**	1,336	250	3	7	3	147 ± 3.5	156 ± 4.1	106 ± 4.3	7 ± 1.9
WS and SF (N,ρ)	S4.e.	**Random, $\rho = 5\%$ (2.c)**									
WS and SF (N,ρ)	S4.f.	**Random, $\rho = 10\%$**	6,301	250	0	0	1	163 ± 3.5	215 ± 2.3	104 ± 3.4	5 ± 1.6
WS and SF (N,ρ)	S4.g.	**Scale-free, $\rho = 1\%$**	750	250	0	34	74	157 ± 2.9	129 ± 3.1	121 ± 4.4	74 ± 0.1
WS and SF (N,ρ)	S4.h.	**Scale-free, $\rho = 2\%$**	1,250	250	0	35	48	171 ± 2.7	147 ± 3.5	121 ± 4.2	48 ± 0.1
WS and SF (N,ρ)	S4.i.	**Scale-free, $\rho = 5\%$ (2.d)**									

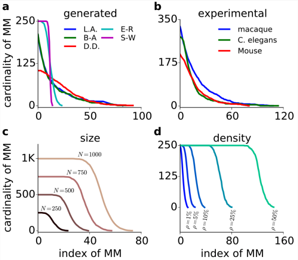
b. Unconditioned pruning

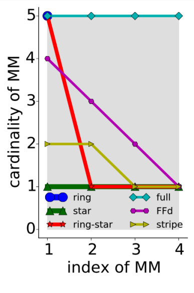
	Fig.	Network	nE	nV	$n(d<3)$	nCN_0	$ePDI$ (in %) ord.	out.	rnd.	rev.
Social	4, 2.g.	**Google +**[76]	1,506,896	211,187	51,680	121,867	78.50	42.52	31.51	5.29
Social	4	**Amazon**[79]	1,234,877	262,111	8,458	12,469	48.78	49.06	27.94	10.27
Social	4	**EU-email**[81]	420,045	265,214	245,791	248,706	92.78	43.08	40.82	2.32
Social	4	**Peer-to-peer**[80, 81]	147,892	62,586	46,227	38,520	23.09	49.70	13.63	5.53
Social	4	**Wikipedia vote**[82]	103,689	7,115	4,736	2,960	60.76	23.11	9.03	1.16
Social	4, 2.h.	**Facebook**[77]	88,234	4,039	568	256	62.67	47.25	16.35	1.97
Social	4, 2.i.	**Airport**[78]	30,501	2,939	872	1,323	73.25	42.49	26.81	3.39

Cardinality curve

Extracting MM reserves







$$E = \bigcup_{k=1}^{l} E_k,$$

where $\quad E_1 \leq_{MM} E$

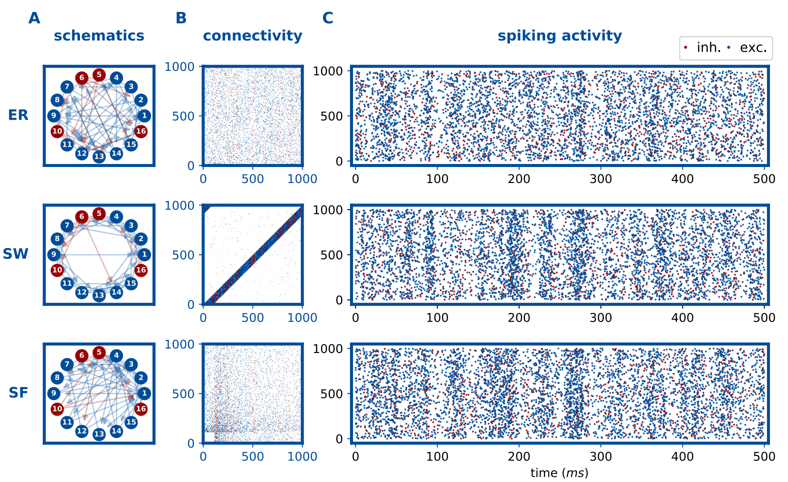
$$E_k \leq_{MM} E - \bigcup_{w=1}^{k-1} E_w$$

Degeneration on Spiking Neural Networks

Parent networks

The three domains of networks before degeneration

(N=1000, ρ=10%, nExc=800, nInh = 200, wInh = -g*wExc)



Quantifying network properties

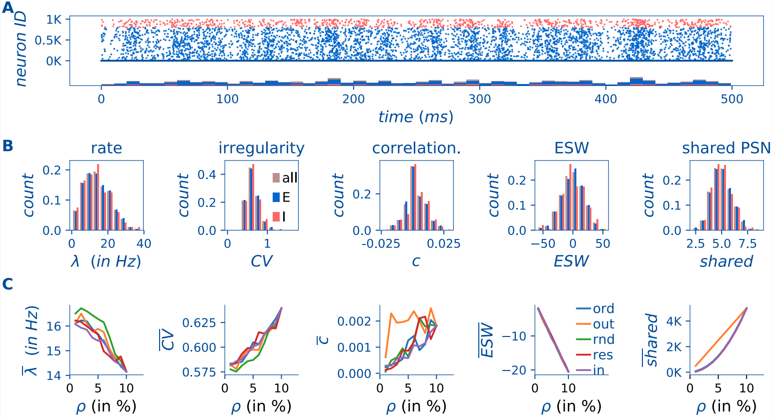
Functional and structural estimates

- ☐ Dynamical features:
  Rate, Variability, Regularity, Correlation, Synchrony, ...
- ☐ Structural features:
  Degree, Effective synaptic input, Shared input, ...
- ☐ Measures of topology
  - **randomness**:
    comparing the degree distribution with random networks of the same size and density.
  - **small-worldness**:

$$\phi = \sqrt{1 - \frac{\Delta_C^2 + \Delta_L^2}{2}}, \quad \text{where} \quad \boldsymbol{\Delta}_C = \frac{C_l - C_o}{C_l - C_r} \quad \text{and} \quad \boldsymbol{\Delta}_L = \frac{L_o - L_r}{L_l - L_r}$$

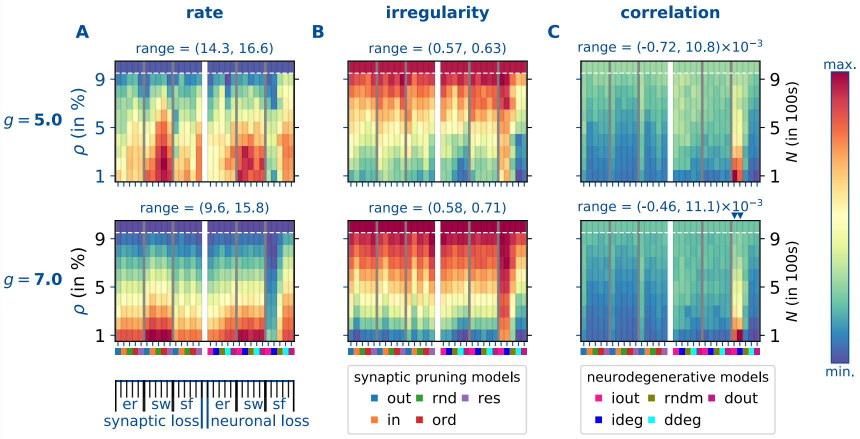
  - **scale-freeness**:
    comparing the degree distribution if it follows power-law and with exponent bounded between 2 and 3.

The estimates for each network is done as follows:



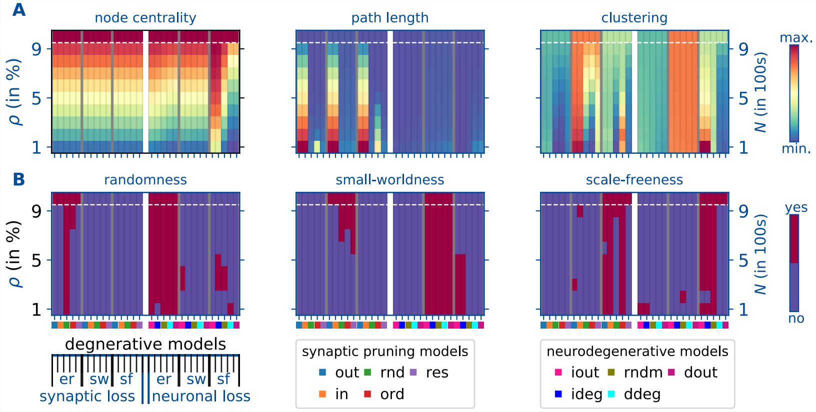
Degeneration on dynamics

Synaptic and/or neuronal death on dynamics



Degeneration on network structure

Synaptic and/or neuronal death on structure



Structure-Dynamics Correlation

Structure-Dynamics Correlation

Correlation between structure and dynamics

Effective synaptic weight correlates well with dynamical estimates