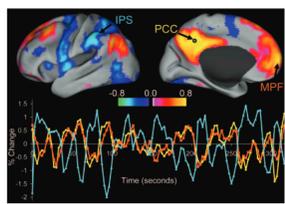


## Introduction

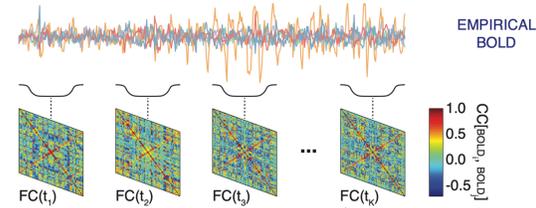
- Functional connectivity of the resting state describes patterns of correlation between distant regions working together
- Usually described as static, time-averaged network but it changes relentlessly through time as described by **dynamic Functional Connectivity (dFC)** analyses
- dFC fluidity is not only ubiquitous, it serves as **biomarker** of efficient cognitive performance and tracks different conditions (aging, pathology...)



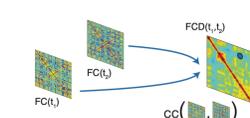
Fox et al. (2005)

The Connectome: Time-Varying Connectivity Networks as the Next Frontier in fMRI Data Discovery

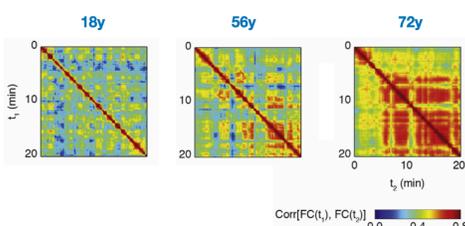
Calhoun et al. (2015)



EMPIRICAL BOLD



Recurrence analyses to detect structure in temporal variability



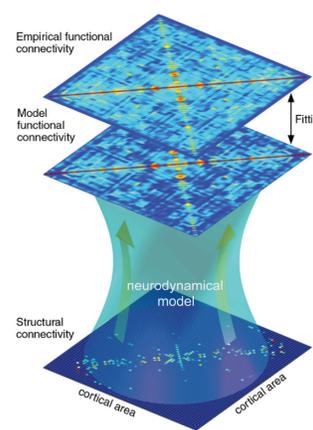
Example: dFC slows down with aging or sleep deprivation, correlating with degraded cognitive assessments

Battaglia et al. Neuroimage (2020)  
Lombardo et al. Neuroimage (2020)

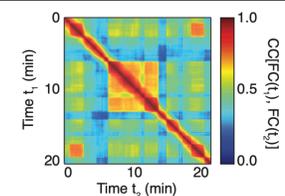
Functional connectivity supposedly reflects **inter-regional interaction and exchange of information**. If the aim of establishing FC is to enable communication why not generating a stable functional network infrastructure, optimized for information exchange efficiency?

In other words, why dFC should be "d" ?

## "Why" as "how to": dFC is naturally occurring



dFC recurrence matrix from simulated rs  
Hansen et al. Neuroimage (2015)



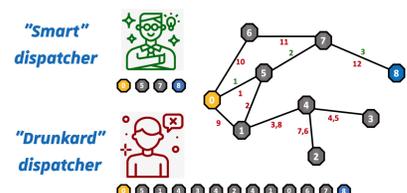
- Mean-field models of whole brain dynamics can be used to emulate resting state dynamics
- Simulations spontaneously give rise to a rich dFC because of the **sampling of an internal repertoire of dynamical states** near a critical instability
- Thus dFC is "d" "why"... it would be unlikely to be otherwise

## "Why" as "what for": dFC as an asset for info flow?

- Consider **FC networks as a communication infrastructure system** on which some unspecified "information" can flow moved by some unspecified dispatcher process. Information can move over a **static graph of FC** but also over a **temporal graph of dFC**
- Suppose that "keeping on" a **FC link at a certain time has some cost**, e.g. metabolic cost of high synchrony. We use **binary undirected networks** for simplicity (one link "on" at time  $t = \text{cost}$  1; Tot cost = tot number of links over space and time)

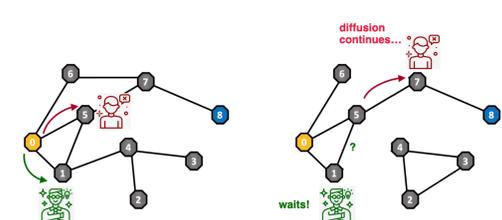
If the system disposes of a **finite amount of resources**, how should it better spend them to dispatch info efficiently? By creating **fewer but long-lasting optimized links**? or, **at equal cost and duration, more transient links**? Is there an **optimum tradeoff** ?

- We study **two strategies for info dispatch** on static and temporal graphs: the **"smart" scenario** corresponds to an **upper bound**, the **"drunkard" scenario** to a **chance expectation**



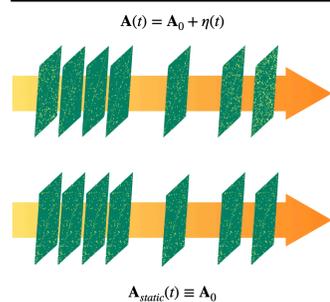
A smart dispatcher always chooses the **shortest path** to move info from a node to another (here "0" to "8")  
A drunkard dispatcher performs a **random walk** on the graph until the target is found

Even more complicated on a temporal net: **Time-respecting paths!**



diffusion continues...  
good paths are not necessarily taken and may also disappear  
uses shortcut at right time at right place

## Toy dynamic Functional Connectivity: random temporal nets



Toy Temporal network  $A(t)$  given by **random fluctuations around a common reference  $A_0$**  (Erdős-Renyi ensemble) Specified by an average degree  $k$  and by a probability  $p_{rew}$  of random rewiring

"Static temporal" network  $A_{static}(t)$  given by the reference matrix  $A_0$  repeated through time.  $A(t)$  and  $A_{static}(t)$  are generated to have **same cost**

For both "smart" and "drunkard" scenarios, we monitor two properties to **quantify info dispatch efficiency**, as a function of average network frame degree, i.e. **for increasing network cost**.

**Irrigation**, i.e. the **amount of nodes reachable by info** initially at a source node over a given horizon of time ( $T \sim 10^3$  frames, equivalent to empirical rs sessions below)

**Impermeability**, i.e. the **resistance to information diffusion** measured as the **time needed for info to reach a target node**, averaged all possible targets (targets that are not reached over the observation time  $T$  contribute a **penalty** of  $T+1$ )

### Analytical understanding

$$B(K) = \prod_{i=0}^{K-1} A_{t+i} B_{ij}(K)$$

$$P_i(i, j) = \frac{A_i(i, j)}{\sum_j A_i(i, j)}$$

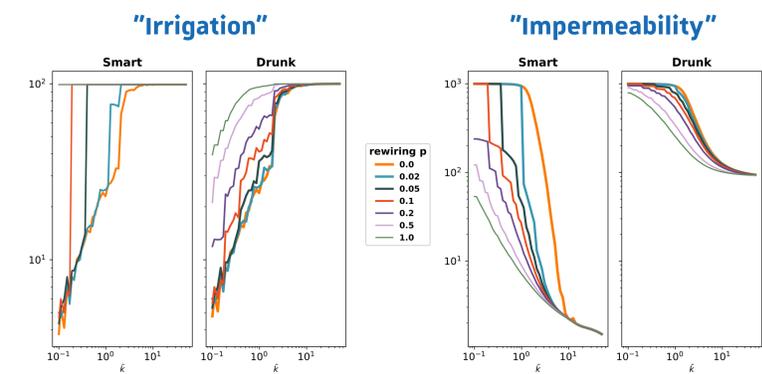
$$\Omega(K) = \prod_{i=0}^{K-1} P_{t+i} \Omega_{ij}(K)$$

For SMART dispatch, **time-ordered products** of instantaneous adjacency matrices describe the existence of time-respecting paths. The smallest  $K$  for which  $B_{ij}$  is not zero is the fastest SMART path

$$MFPT(i, j) = \sum_{k=1}^T k \cdot \Omega_{ij}(k) (1 - \Omega_{ij}(k-1))$$

The **mean first passage time** in node  $j$  of DRUNKARD paths starting in  $i$  can be computed as a probability-weighted average

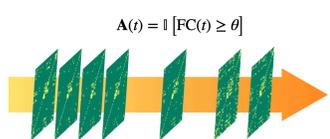
Improvement of info diffusion by dynamic rewiring are given by the **aggregate network reaching percolation threshold**



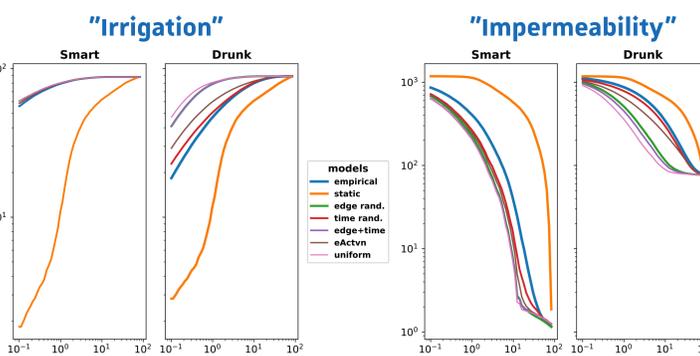
Increasing network density, **irrigation increases**. Adding **random link rewiring boosts overall irrigation**

Increasing network density, **impermeability decreases**. Adding **random link rewiring reduces overall impermeability**

## Empirical dynamic Functional Connectivity: human rs fMRI



A binary temporal network  $A(t)$  is constructed from an **empirically measured dFC stream** from 20 min of human BOLD rs fMRI (time window 20 s, data from Human Connectome Project, 99 subjects)  
Application of a **sharp threshold  $\theta$**  to control the obtained average degree.  $A_{static}(t)$  is constructed from an **average temporal network frame**, rebinarized to obtain a temporal network with the **same cost**



Static dFC has the **lowest Irrigation** among all considered models.

Static dFC has the **highest impermeability** among all considered models.

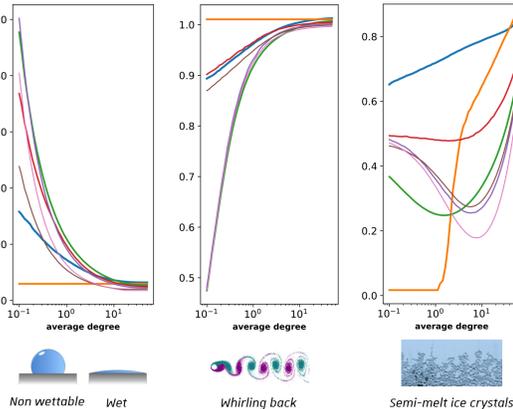
Empirical dFC has **worse irrigation than more random** dynamic models.

Empirical dFC is **more impermeable than more random** dynamic models.

We compare **empirical dFC streams** with an equal cost **static** version. In addition we have a variety of **null models**. In **edge-randomized** link density is maintained frame-by-frame but links are randomly rewired, while in **uniform** every frame is regenerated from scratch with a homogeneous static-like link density in every frame. In **time randomized**, frames are kept identical but time-shuffled. In **"edge+time"**, after time shuffling, edges are rewired within each frame. Finally in **edge-activation**, every link is randomly generated with a link-specific activation probability matched to empirical.

**Empirical dFC** dispatches information ("integration") **more efficiently** than its **static** counterpart. However, it **does it less well** than the other **less structured null models**.  
**What else is empirical dFC optimizing besides integrative dispatch?**

### "Wettability" "Eddying" "Slushing"



We define **wettability** as the time needed for information leaving a node to flow back to it, i.e. to stick to a place like a droplet that does not spread. **Empirical dFC is less wetttable** than most other dynamic models

Related to wettability, we define **eddyding** as the probability that future neighbors of a node's neighbors at a reference time, still include nodes from the initial reference neighbor set, i.e. that the info flow tends to whirl back to a local neighborhood (high cross-time Jaccard index). **Empirical dFC displays more eddyding** than most other dynamic models.

We also track **temporal clustering**, i.e. the tendency to have high clustering when overlaying temporally close frames, even when individual frames may be more poorly clustered. Like in an icy slush, where crystals are melting and refreezing fluidly. **Empirical dFC has the highest temporal clustering** of all dynamic models.

**Empirical dFC** has the **highest capacity to retain and mix information locally** among all tested dynamical models. Other dynamic models may be more integrative, but, unlike empirical dFC, they lack these **"segregation"** properties.

### Some open questions

- Can we **understand analytically** the scaling of irrigation curves ?
- Can we construct a **null-model** capturing more precisely **both integrative and segregative** properties of empirical dFC? Or a **virtual brain model**?
- Are the frame-by-frame fluctuations in network density of empirical dFC interpretable as an **avalanching phenomenon**? Power-laws?
- Are our info flow features useful as **biomarkers of cognitive (dys)function**?

## Conclusions

- The **"d" in dFC** confers **superior information dispatch properties** in terms of both number of irrigated nodes and time needed to receive a broadcasted information. Such **integrative advantages are more evident** when networks are more diluted, i.e. when creation and maintenance through time of a **FC link have a high cost**
- Between **static FC and more random null models**, **empirical dFC** exhibits a **complex tradeoff** between advantages for **integration** conferred by dynamicity and the simultaneous preservation of **segregative properties**.